PHOTONIC BAND STRUCTURES FOR A CLASS OF 2D PERIODIC DIELECTRIC MATERIALS

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ABSTRACT

In recent years, there has been extensive interest in the development of artificial electrical or optical materials. By tailoring the material electrical characteristics, one is able to control the flow of electromagnetic waves from microwave to optic frequencies. This paper presents the guided wave properties of a class of two dimensional photonic crystals made of periodic dielectric rods. An efficient finite difference method is developed for the calculation of propagation constants of guided or evanescent waves in an arbitrary direction (in-plane or out-of-plane propagation). The emphasis is on the existence of photonic bandgaps within which the in-plane propagation is prohibited. Possible applications of photonic bandgap materials are also discussed.

I. INTRODUCTION

Wave propagation in periodic structures has been an important and interesting subject to electromagnetic society for many decades. Artificial dielectrics composed of infinite arrays of periodic conductors had been proposed for microwave lens applications [1]. Periodically loaded waveguides have found applications in a variety of devices such as traveling-wave tubes, filter networks, and surface waveguiding devices [2]. Planar printed metallic elements periodically distributed over the surface of a dielectric layer have been used in frequency selective surfaces [3] and integrated phased array antennas [4]. Light interaction with dust and rain drops and x-ray diffraction from crystals are also of practical interest related to wave interaction with randomly distributed periodic structures [5]. A common feature of periodic structures is the existence of frequency bands where electromagnetic waves are highly attenuating and do not propagate. In analogy to an electrical crystal where periodic atoms or molecules may present a bandgap prohibiting electron propagation, a photonic crystal is made of macroscopic dielectrics periodically placed (or embedded) within a surrounding medium. The periodic nature of the structure may introduce photonic band gap (PBG) within which photons (wave propagation) are forbidden in certain directions.

The concept of photonic band gap introduced by Yablonovitch in 1987 [6] for semiconductor lasers and for photonic applications [7-8] has stimulated significant research interest among physicists. A summary of the up-to-date research on this subject can be found in [9]. Photonic crystal research is motivated by the consensus that optical, or more general, electromagnetic technologies may benefit from photonic crystals in a similar way electronic technology benefits from semiconductors.

In the past, most of the two dimensional (2D) photonic crystals analyzed are for circular columns and with a plane wave expansion method. In this paper, a class of 2D photonic crystals made of periodic arrays of irregular rods are investigated with a finite difference method. The structure can be made with conventional machine tools in the centimeter range and with micromachine technique in the micron range. The guided wave modes within such a structure are identified with emphasis on the search of a complete bandgap where wave propagation is prohibited in all the in-plane directions.

For 2D photonic crystals, the geometry is uniform along the longitudinal ($\hat{z}$) direction and periodic in the transverse (in-plane) direction with lattice constants a and b. Unit cells of examples of photonic crystals under investigation are shown in Figure 1. The geometry is somewhat similar to periodic arrays of dielectric waveguides. However, we are interested in wave propagation in the transverse plane where, due to the periodic property, wave propagation may be prohibited.

II. FINITE DIFFERENCE ANALYSIS OF TWO-DIMENSIONAL PERIODIC STRUCTURES

An efficient finite difference method for periodic structures is developed for the computation of propagation constants of guided wave propagating in arbitrary directions. Field analysis of waveguide structures usually deals with Helmholtz equations, in terms of two of the six field components. In finite element or finite difference method, H-field formulation is more preferable due to the fact that magnetic field is continuous. In this analysis, the H_x and H_y formulation is employed to avoid possible spurious-mode problem [10].

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The Helmholtz equations of the pertinent problem are

$$\frac{\partial^2 H_x^{(i)}}{\partial x^2} + \frac{\partial^2 H_x^{(i)}}{\partial y^2} + (k_i^2 - \beta_z^2) H_x^{(i)} = 0 \qquad (1)$$

and

$$\frac{\partial^2 H_y^{(i)}}{\partial x^2} + \frac{\partial^2 H_y^{(i)}}{\partial y^2} + (k_i^2 - \beta_z^2) H_y^{(i)} = 0 \quad , \qquad (2)$$

where β_z is the phase constant in the $\hat{z}$ direction, $k_i = k_0\sqrt{\varepsilon_i}$, k_0 is the free space wave number, and ε_i is the dielectric constant in region i. In finite difference method, the unit cell $0 \le x \le a$ and $0 \le y \le b$ is divided into many rectangular grids. The common point of four adjacent grids is a central node where H_x and H_y are related to those of the adjacent nodes through a five-point finite difference equation. The five-point finite difference form of a Helmoholtz equation for four connected grids with different dielectric constants can be found in Reference [10]. A finite difference equation is to relate the fields at one central node to the four adjacent nodes. If there are M nodes in each side of a unit cell, there are $2M^2$ finite difference equations with $2M^2$ unknowns. For the central nodes at the boundary, some of the adjacent nodes are out of the unit cell and can be brought back into the unit cell utilizing the periodic nature of the structure.

$$\Phi(x+a, y+b) = e^{-j\beta_x a - j\beta_y b}\Phi(x,y) \qquad (3)$$

where $\Phi(x,y)$ is any field component, β_x and β_y are the phase constants in the $\hat{x}$ and $\hat{y}$ directions, respectively. The $2M^2$ finite difference equations form a set of linear homogeneous equations or matrix equations. The eigenvalue equation is obtained by setting the matrix determinant to zero. The roots of the eigenvalue equation are frequencies for a given the phase constants of the waves propagating within the photonic crystals.

In numerical implementation, the direct approach using Gaussian elimination is not practical. For the bisection method of finding the roots, it often require many iterations. For a unit cell with 400 grids (20 divisions in each direction), the matrix dimension would be 882. The required computer memory and time for each iteration are enormous. An alternate approach is to utilize the matrix sparsity in the QR procedure. The matrix is band-diagonal except the last few rows and columns due to the periodic properties of the finite-difference cells. A modified QR procedure to deal with this kind of matrix is developed. For $2M^2$ nodes, the matrix dimension is reduced to 4M-2 and the required Gaussian elimination procedure is reduced to $(4M-2)^2 + (4M-2)!$.

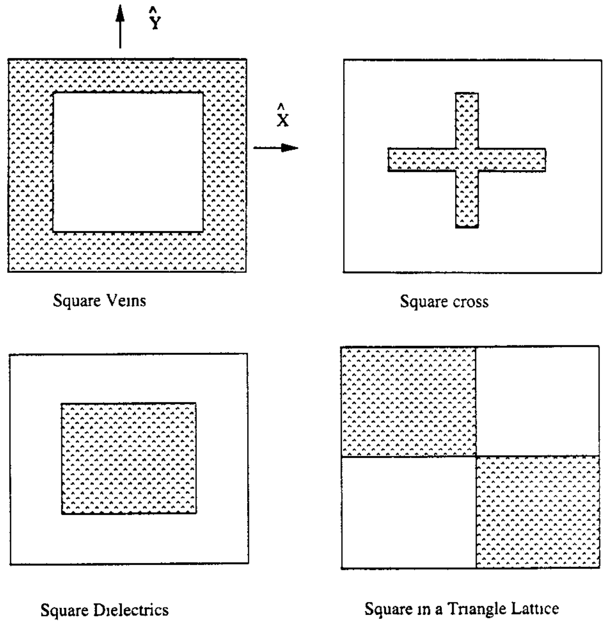

Figure 1. Cross sections of unit cells of examples of two dimensional photonic crystals. Structures are uniform in the z direction.

III. RESULTS FOR GUIDED WAVES AND PHOTONIC BAND GAPS

It is known for dielectric waveguides that propagating waves are hybrid. However, for in-plane propagation (wave vector in the transverse plane), the guided wave modes can be decoupled into either TE or TM modes (to the z) with no z variation. The coordinates are shown in Figure 1. The finite difference method is general enough to deal with a variety of irregular photonic crystal structures. The accuracy and validity of the finite difference analysis are checked in excellent agreement with the results shown in Reference [9] where a plane wave expansion method is used.

It is known that there may exist photonic band gap for 2D periodic material structures where all the in-plane propagation is prohibited. The engineering applications of such photonic band-gap materials would result in devices with new functionalities, not possible otherwise, such as optical shield, low-loss photonic waveguides, optical high-Q resonators, optical transducers, directional couplers, etc.. Signature alternation and identification, ultra-high gain antennas, laser emission and protection are some other applications.

The photonic band structure for periodic dielectric squares is shown in Figure 2 for both TE and TM modes. The horizontal axis in Figure 2 is for the phase constants of guided wave modes in various directions. Due to the symmetric and periodic properties, only the shade region in the Brillouin zone (the wave number space or the reciprocal lattice) is irreducible. For example, the phase constants in the $\hat{x}, \hat{y}, -\hat{x}$, and $-\hat{y}$ direction propagation are the same. It is also seen from Figure 2 that there exists a wide band (photonic gap) for TM waves (but not for TE waves) where wave propagation is prohibited in all directions. In contrast, air squares surrounded by high dielectrics (dielectric veins) result in a wide band gap for TE waves but not for TM waves. The location and the width of the band gap are determined by the dielectric constant and the filling factor (percentages of the implant occupancy). Within the band gap region, the propagation modes become pairs of complex modes [11] that do not carry energy.

The photonic band structure for periodic dielectric crosses is shown in Figure 3. One can also treat this geometry as broken dielectric veins with air gap 13% of the cell width (cross length extends 87% of the cell). It is seen that there exist photonic band gaps for both TE and TM waves. This is because dielectric veins tend to have TE band gap, while dielectric implants tend to have TM band gap. Dielectric crosses with long arms resemble both cases. It is found that the band gap width is mainly determined by the air gap. Usually, the smaller the air gap is (more like a vein), the larger the TE gap and the smaller the TM gap would be. For the case shown in Figure 3, if the cell size is 1 cm, the TM band gap is from 9 GHz to 9.4 GHz, and the TE band gap is from 13.3 GHz to 13.9 GHz.

An example of band structure for the oblique direction of propagation (out-of-plane) is shown in Figure 4 for crystals with square dielectrics. The z direction phase-constant is assumed as 1/a. It is noted that, due to the homogeniety in the logitudinal direction, there is no photonic gap. The geometry of the structure is identical to that for Figure 2. Several interesting observations are found from the comparison of these two Figures. For out-of-plane propagation, the modes are hybrid and only the first two fundamental modes (not the rest) are very different from those for in-plane-propagation. It is seen from Figure 4 that there exists transverse mode cut-off. This is no surprise since if we specify the longitudinal phase constant, the frequency can not be arbitrary. .The other explanation is that if we specify the longitudinal wave number, equivalently, we are dealing with the higher order modes of parallel plates containing photonic crystals. Cut-off frequency does exist in parallel-plates for non-TEM modes.

IV. ACKNOWLEDGMENT.

The author would like to thank Professor N.G. Alexopoulos at UCLA for helpful suggestions.

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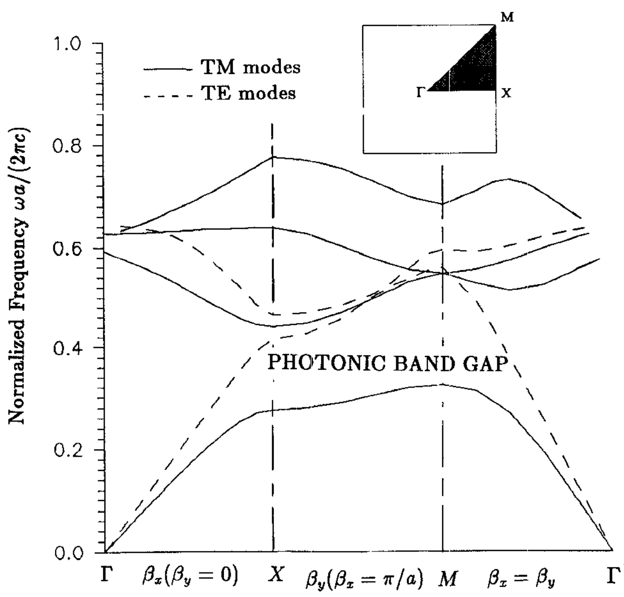

Figure 2. The photonic band structure for the first few modes of dielectric squares $\varepsilon_r = 8.9$ surrounded by air. The square length: 0.3545a. The horizontal axis is for wave numbers in various directions. (Γ, X, M are symmetric points in Brillouin zone shown in the inset).

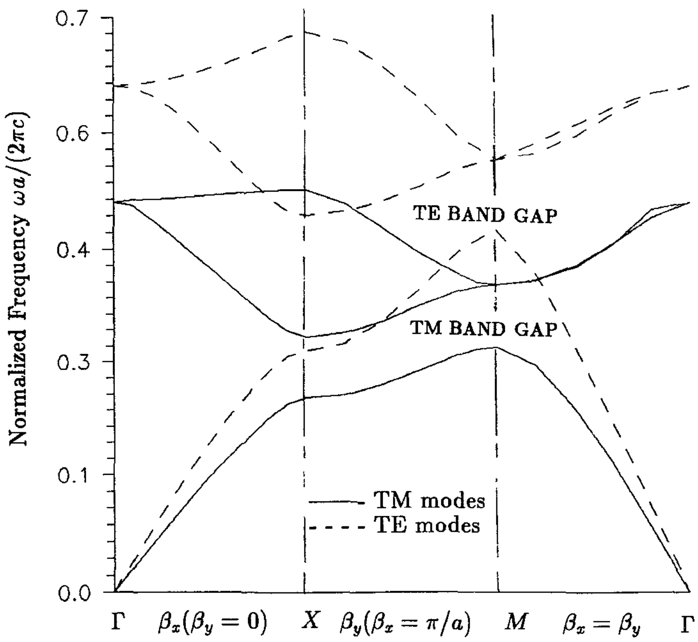

Figure 3. The photonic band structure for the first few modes of dielectric crosses $\epsilon_r = 8.9$ surrounded by air. The cross length: $0.87a$, width: $0.181a$.

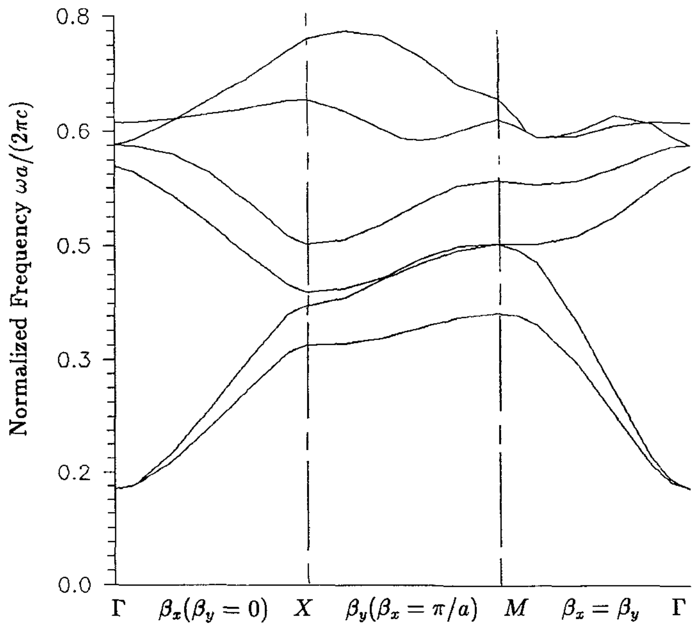

Figure 4. The photonic band structure for the first six modes of dielectric squares $\epsilon_r = 8.9$ surrounded by air.
The square length: $3545a$. Out-of-Plane Propagation $\beta_z = 1/a$.